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Gain-scheduling Control of T-S Fuzzy Systems with Actuator Saturation

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Abstract. This paper presents a gain-scheduling output feedback control design method for T-S fuzzy systems with actuator saturation. Different from existing control design methods for T-S fuzzy systems, the basic idea of the proposed approach is to transform the T-S fuzzy model with saturation nonlinearity into the form of linear fractional transformation (LFT). Instead of commonly used fuzzy controllers, a gain-scheduled output feedback controller in the LFT form is introduced to stabilize the saturated T-S fuzzy system with guaranteed H_∞ performance. The problem of establishing regional stability and performance of the closed-loop nonlinear system are tackled by using robust control techniques. As a result, the conservatism introduced by dealing with the quadratic terms of normalized fuzzy weighting functions can be avoided. The proposed controller synthesis problem is cast as a convex optimization in terms of linear matrix inequalities (LMIs) and can be solved efficiently. An example of balancing the inverted pendulum with bounded actuation is provided to illustrate the effectiveness of the proposed design method.

Keywords: T-S fuzzy systems, actuator saturation, gain-scheduling control, linear fractional transformation

1. Introduction

Actuator saturation commonly exists in control engineering because of the impossibility for any actuator to provide unlimited actuation magnitude. Due to the existence of saturation nonlinearity can degrade systems performance or even cause instability of closed-loop systems, systems with actuator saturation have raised great research interest in the communities of both control theory and engineering [1,2,3]. For a linear system with input saturation, the analysis and synthesis conditions have been developed extensively (see [4] and the references therein). Semi-global stabilization or global stabilization can be achieved for the open-loop stable systems [5,6,7]. As to the systems,

for which only local stabilization can be achieved, the problem of estimating the domain of attraction and designing controller to enlarge the domain of attraction have attracted great attention [8,9,10].

Another common but more difficult control problem is the stabilization of nonlinear systems with actuator saturation. Among the research schemes on this issue, the fuzzy control method, which is well developed for many control design problems due to its excellent approximation ability of nonlinear dynamics (such as control design for stochastic systems [11,12], network-based systems [13,14], affine dynamic systems [15], hyperbolic PDE systems [16] et al.), was applied to deal with the stabilization of nonlinear systems with actuator saturation [17,18,19,20,21,22,23,24]. In [17], the low-gain design approach is used to constrain the magnitude of the control output. Following the method

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of dealing with actuator saturation in [3], a less conservative approach based on convex hull representations has been proposed in [18] to cope with the saturation nonlinearity in nonlinear systems. In [19], the saturation function is captured by a specific nonlinear saturation sector and a design approach is proposed which requires less number of linear matrix inequalities (LMIs). However, all the aforementioned results only concern with the state feedback controllers. A fuzzy observer-based controller for nonlinear distributed parameter systems with control limitations is investigated in [20]. The input saturation is represented by a polytopic model and a static output feedback controller design method is presented in [21]. Moreover, a dynamic output feedback fuzzy controller is considered in [22], in which both the amplitude saturation and the rate limitation are taken into consideration. Furthermore, the authors of [23] deal with the time-delay fuzzy systems with actuator saturations, and propose an anti-windup control approach to maximize the size of the estimated domain of attraction.

In this work, the issue of stability and H_∞ performance of the T-S fuzzy systems with actuator saturation is reconsidered. To this end, a gain-scheduled output feedback controller in the form of linear fractional transformation (LFT) is proposed to stabilize the saturated T-S fuzzy system with regional H_∞ performance guarantee. More specifically, in order to deal with the saturation nonlinearity, the deadzone function is introduced based on the method in [25]. Thus by algebraic manipulations, the T-S fuzzy model with actuator saturation nonlinearity can be transformed into the form of linear fractional transformation. It can be shown that existing dynamic parallel distributed compensator (DPDC), which is widely used for T-S fuzzy model, is just a special case of the LFT gain-scheduled controller proposed in this article. So a better performance can be obtained with the proposed gain-scheduled controller over the DPDC. By utilizing the LFT mechanism [26,27,28], the synthesis problem is cast as a convex optimization problem in terms of linear matrix inequalities (LMIs) and can be solved efficiently. Moreover, a two-step LMI-based synthesis procedure is proposed to construct the gain-scheduled output feedback controller gains. Finally, An example of balancing the inverted pendulum by saturated control is provided to illustrate the effectiveness of the proposed control approach.

The main contributions of this work can be summarized as follows: First, the proposed method provides a novel approach to optimize the closed-loop per-

formance and maximize the size of operating region for T-S fuzzy systems with input saturated. Second, a method of transforming the T-S fuzzy model into the form of LFT is suggested, which can be a useful connection in applying the powerful robust control techniques to T-S fuzzy systems. Third, the design conditions are simply expressed in terms of LMIs, in such a way, the controller design problem can be effectively solved through convex optimizations.

The reminder of this paper is organized as follows. In Section 2, the T-S fuzzy model with actuator saturation and the method converting it into a LFT model are presented. In Section 3, the gain-scheduled output feedback controller is proposed for saturated fuzzy systems and then the main result is presented. An example of balancing an inverted pendulum on a cart using saturated control input is presented in Section 4, followed by conclusions in Section 5.

The notations used in this paper are quite standard. \mathbf{R} stands for the set of real numbers and \mathbf{R}_+ for the non-negative real numbers. \mathbf{Z}_+ is the set of non-negative integers. $\mathbf{R}^{m \times n}$ is the set of real $m \times n$ matrices. Besides, we use \mathbf{S}^n to denote real, symmetric $n \times n$ matrices, and \mathbf{S}_+^n for positive-definite matrices. A block-diagonal matrix with matrices X_1, X_2, \dots, X_p on its main diagonal is denoted as $\text{diag}\{X_1, X_2, \dots, X_p\}$. In large symmetric matrix expressions, terms denoted as \star will be induced by symmetry. For two integers $k_1, k_2, k_1 < k_2$, we denote $\mathbf{I}[k_1, k_2] = \{k_1, k_1 + 1, \dots, k_2\}$. The space of square integrable functions is denoted by \mathcal{L}_2 , that is, for any $x \in \mathcal{L}_2$

$$\|x\|_2 = \left(\int_0^\infty x^T(t)x(t)dt \right)^{\frac{1}{2}} < \infty.$$

2. T-S fuzzy model and preliminaries

A T-S fuzzy model representing the dynamics of a nonlinear plant subject to actuator saturation will be considered in this paper. The i th rule of the T-S fuzzy model can be described by the following linguistic rule,

$$\begin{aligned} &\text{IF } z_1 \text{ is } M_i^1 \text{ AND } \dots z_r \text{ is } M_i^r, \\ &\text{THEN } \begin{cases} \dot{x} &= A_i x + B_{1,i} d + B_{2,i} \text{sat}(u), \\ e &= C_{1,i} x + D_{11,i} d + D_{12,i} \text{sat}(u), \\ y &= C_{2,i} x + D_{21,i} d, \end{cases} \end{aligned} \quad (1)$$

where $i \in \mathbf{I}[1, m]$, m is a positive integer which denotes the number of fuzzy rules; $z_j, j \in \mathbf{I}[1, r]$, is the premise or antecedent variable which is measurable; and r is a positive integer representing the total number of premise variables; M_i^j is a fuzzy or linguistic term, which can be quantified by membership functions, including triangular, trapezoid, and Gaussian-shaped membership functions and so on. Besides, the variable $x \in \mathbf{R}^n$ is the plant state; $u \in \mathbf{R}^{n_u}$ is the control input; $d \in \mathbf{R}^{n_d}$ is the exogenous input, possibly including disturbance, measurement noise or reference signals; $y \in \mathbf{R}^{n_y}$ is the measurement output and $e \in \mathbf{R}^{n_e}$ is the controlled output. Throughout this research, it is assumed that $(A_i, B_{2,i})$ is stabilizable and $(C_{2,i}, A_i)$ is detectable for all $i \in \mathbf{I}[1, m]$. $\text{sat}(\cdot)$ is a vectorized saturation function with the saturation levels given by a vector $\bar{u} \in \mathbf{R}^{n_u}$, $\bar{u}_i > 0, i \in \mathbf{I}[1, n_u]$. More specifically, $\text{sat}(u_i) = \text{sgn}(u_i) \min\{\bar{u}_i, |u_i|\}$. For disturbance attenuation, we are mainly concerned with a class of energy-bounded disturbances

$$\mathcal{W}_s = \{d : \mathbf{R}_+ \rightarrow \mathbf{R}^{n_d}, d \in \mathcal{L}_2, \int_0^\infty d^T(\tau)d(\tau)d\tau \leq s^2\},$$

in which s is a given positive scalar indicating energy level.

For subsequent development, we will first transform the T-S fuzzy model into LFT form, described by

$$\begin{bmatrix} \dot{x} \\ q \\ e \\ y \end{bmatrix} = \begin{bmatrix} A & B_0 & B_1 & B_2 \\ C_0 & D_{00} & D_{01} & D_{02} \\ C_1 & D_{10} & D_{11} & D_{12} \\ C_2 & D_{20} & D_{21} & 0 \end{bmatrix} \begin{bmatrix} x \\ p \\ d \\ u \end{bmatrix}, \quad (2)$$

$$p = \Theta q, \quad (3)$$

where $q, p \in \mathbf{R}^{n_p}$ are the plant pseudo-output and pseudo-input, respectively. Note that the pseudo-input and pseudo-output connecting to the uncertain matrix Θ are internal signals to separate the parametric uncertainty from the nominal plant part. Θ is a time-varying structured parameter matrix in the form of

$$\Theta = \left\{ \text{diag}\{\theta_1 I_{r_1}, \dots, \theta_s I_{r_s}\} : |\theta_j| \leq 1, j \in \mathbf{I}[1, s] \right\},$$

where $\sum_{j=1}^s r_j = n_p$, and θ_j is the scheduling parameter available in real time for control use.

By quantifying the linguistic terms and using defuzzification method, the analytical formulation of the

T-S fuzzy plant can be obtained as below

$$\begin{cases} \dot{x} &= \sum_{i=1}^m g_i(z) [A_i x + B_{1,i} d + B_{2,i} \text{sat}(u)], \\ e &= \sum_{i=1}^m g_i(z) [C_{1,i} x + D_{11,i} d + D_{12,i} \text{sat}(u)], \\ y &= \sum_{i=1}^m g_i(z) (C_{2,i} x + D_{21,i} d), \end{cases} \quad (4)$$

with

$$g_i(z) = \frac{w_i(z)}{\sum_{i=1}^m w_i(z)}, \quad w_i(z) = \prod_{j=1}^r M_i^j(z_j),$$

where $z = [z_1 \ z_2 \ \dots \ z_r]^T$, and $M_i^j(z_j)$ is the degree of membership of z_j in M_i^j . Since the degree of membership is confined into the closed interval $[0, 1]$, we have the following properties

$$0 \leq g_i(z) \leq 1, \quad \sum_{i=1}^m g_i(z) = 1.$$

By introducing a nominal model, equations (4) can be rewritten as

$$\begin{cases} \dot{x} &= A_0 x + B_{1,0} d + B_{2,0} \text{sat}(u) \\ &\quad + \sum_{i=1}^m g_i(z) [A_{\delta i} x + B_{1,\delta i} d + B_{2,\delta i} \text{sat}(u)], \\ e &= C_{1,0} x + D_{11,0} d + D_{12,0} \text{sat}(u) \\ &\quad + \sum_{i=1}^m g_i(z) [C_{1,\delta i} x + D_{11,\delta i} d + D_{12,\delta i} \text{sat}(u)], \\ y &= C_{2,0} x + D_{21,0} d \\ &\quad + \sum_{i=1}^m g_i(z) (C_{2,\delta i} x + D_{21,\delta i} d), \end{cases} \quad (5)$$

with

$$\begin{aligned} A_0 + A_{\delta i} &= A_i, \\ B_{1,0} + B_{1,\delta i} &= B_{1,i}, \quad B_{2,0} + B_{2,\delta i} = B_{2,i}, \\ C_{1,0} + C_{1,\delta i} &= C_{1,i}, \quad C_{2,0} + C_{2,\delta i} = C_{2,i}, \\ D_{11,0} + D_{11,\delta i} &= D_{11,i}, \quad D_{12,0} + D_{12,\delta i} = D_{12,i}, \\ D_{21,0} + D_{21,\delta i} &= D_{21,i}. \end{aligned}$$

Further introducing the pseudo-input p_f , the pseudo output q_f , and denoting $\theta_{f,i} = g_i(z)$, we obtain

$$\begin{cases} \dot{x} &= A_0 x + B_{1,0} d + B_{2,0} \text{sat}(u) + \sum_{i=1}^m p_{x,i}, \\ q_{x,i} &= A_{\delta i} x + B_{1,\delta i} d + B_{2,\delta i} \text{sat}(u), \\ q_{e,i} &= C_{1,\delta i} x + D_{11,\delta i} d + D_{12,\delta i} \text{sat}(u), \\ q_{y,i} &= C_{2,\delta i} x + D_{21,\delta i} d, \\ e &= C_{1,0} x + D_{11,0} d + D_{12,0} \text{sat}(u) + \sum_{i=1}^m p_{e,i}, \\ y &= C_{2,0} x + D_{21,0} d + \sum_{i=1}^m p_{y,i}, \end{cases} \quad (6)$$

with $p_{x,i} = (\theta_{f,i} I_n) q_{x,i}$, $p_{e,i} = (\theta_{f,i} I_{n_e}) q_{e,i}$, $p_{y,i} = (\theta_{f,i} I_{n_y}) q_{y,i}$. Consequently, the T-S fuzzy plant with actuator saturation can be converted into a more compact form, with the deadzone nonlinearity, i.e., $\text{dz}(u) = u - \text{sat}(u)$, as below

$$\begin{bmatrix} \dot{x} \\ u \\ q_f \\ e \\ y \end{bmatrix} = \begin{bmatrix} A_0 & -B_{2,0} & E_x & B_{1,0} & B_{2,0} \\ 0 & 0 & 0 & 0 & I \\ \Delta_x & -\Delta_u & 0 & \Delta_d & \Delta_u \\ C_{1,0} & -D_{12,0} & E_e & D_{11,0} & D_{12,0} \\ C_{2,0} & 0 & E_y & D_{21,0} & 0 \end{bmatrix} \begin{bmatrix} x \\ p_s \\ p_f \\ d \\ u \end{bmatrix}, \quad (7)$$

$$p_s = \text{dz}(u), \quad (8)$$

$$p_f = \Theta_f q_f. \quad (9)$$

where $n_q = n + n_e + n_y$ and

$$\begin{aligned} q_f^T &= [q_{x,1}^T \ q_{e,1}^T \ q_{y,1}^T | \cdots | q_{x,m}^T \ q_{e,m}^T \ q_{y,m}^T], \\ p_f^T &= [p_{x,1}^T \ p_{e,1}^T \ p_{y,1}^T | \cdots | p_{x,m}^T \ p_{e,m}^T \ p_{y,m}^T], \\ E_x &= [I_n \ 0 \ 0 | \cdots | I_n \ 0 \ 0]_{n \times (mn_q)}, \\ E_e &= [0 \ I_{n_e} \ 0 | \cdots | 0 \ I_{n_e} \ 0]_{n_e \times (mn_q)}, \\ E_y &= [0 \ 0 \ I_{n_y} | \cdots | 0 \ 0 \ I_{n_y}]_{n_y \times (mn_q)}, \\ \Delta_x^T &= [A_{\delta 1}^T \ C_{1,\delta 1}^T \ C_{2,\delta 1}^T | \cdots | A_{\delta m}^T \ C_{1,\delta m}^T \ C_{2,\delta m}^T], \\ \Delta_d^T &= [B_{1,\delta 1}^T \ D_{11,\delta 1}^T \ D_{21,\delta 1}^T | \cdots | B_{1,\delta m}^T \ D_{11,\delta m}^T \ D_{21,\delta m}^T], \\ \Delta_u^T &= [B_{2,\delta 1}^T \ D_{12,\delta 1}^T \ 0 | \cdots | B_{2,\delta m}^T \ D_{12,\delta m}^T \ 0], \\ \Theta_f &= \text{diag}\{\theta_{f,1} I_{n_q}, \cdots, \theta_{f,m} I_{n_q}\}, \end{aligned}$$

with $0 \leq \Theta_f \leq I$.

With all the above transformations, the T-S fuzzy plant with actuator saturation (1) has been reformulated into a LFT model (7)-(9). Now, the property as described in the following lemma will be used to deal with the deadzone nonlinearity.

Lemma 1 ([25]). Let $t(x) = Tx$ be a linear map and suppose $T_i x \in [-\bar{u}_i, \bar{u}_i]$, where T_i denotes i th row of the matrix T . For any u_i , we have $\text{sat}(u_i) \in \text{Co}\{u_i, T_i x\}$ and $\text{dz}(u_i) = \theta_{s,i}(u_i - T_i x)$ for some $\theta_{s,i} \in [0, 1]$.

From Lemma 1, one can capture the nonlinear relation $p_s = \text{dz}(u)$ by the inequality

$$p_s^T \Lambda_1 \left(u - [H_1 \ H_2] \begin{bmatrix} x \\ x_k \end{bmatrix} - p_s \right) \geq 0, \quad (10)$$

under the regional constraint

$$\left| [H_1 \ H_2]_\ell \begin{bmatrix} x \\ x_k \end{bmatrix} \right| \leq \bar{u}_\ell, \quad (11)$$

where $\Lambda_1 > 0$ is an arbitrary diagonal matrix; H_1, H_2 are the linear maps of x and x_k , which are matrices satisfying condition (11); x_k is the state of the controller to be designed; the subscript $\ell \in \mathbf{I}[1, n_u]$ specifies the ℓ th row of the corresponding matrix.

3. Gain-scheduled dynamic output feedback controller design

In this section, we will study a gain-scheduling dynamic output feedback control problem for the T-S fuzzy systems with actuator saturation (1). Our objective is to synthesize a gain-scheduled controller, the form of which will be detailed subsequently, to make the equilibrium of the saturated fuzzy system asymptotically stable when d vanishes and to render the regional L_2 gain from the exogenous input d to the controlled output e less than a prescribed constant γ when $d \in \mathcal{W}_s$.

3.1. Gain-scheduled dynamic output feedback controller form

Note that the T-S fuzzy plant with actuator saturation has been transformed into an LFT form with Θ_f as its scheduling matrix. Parallel to the parameter-dependency fashion of the saturated LFT plant, a LFT gain-scheduled controller is proposed as

$$\begin{bmatrix} \dot{x}_k \\ q_{k,f} \\ u \end{bmatrix} = \begin{bmatrix} A_k & B_{k0,f} & B_{k0,s} & B_{k2} \\ C_{k0} & D_{k00,f} & D_{k00,s} & D_{k02} \\ C_{k2} & D_{k20,f} & D_{k20,s} & D_{k22} \end{bmatrix} \begin{bmatrix} x_k \\ p_{k,f} \\ \text{dz}(u) \\ y \end{bmatrix}, \quad (12)$$

$$p_{k,f} = \Theta_f q_{k,f}, \quad (13)$$

where the variables $x_k, q_{k,f}, p_{k,f}$ have the same dimensions as x, q_f, p_f , respectively. The matrix Θ_f can be measured online through membership functions. The additional input $dz(u)$ is introduced to deal with the saturation nonlinearity and it can also be determined online. The matrices A_k, \dots, D_{k22} in equation (12) are controller gains to be synthesized.

This controller has rational functional dependency on the scheduling parameter Θ_f (i.e. $g_i(z)$), and is more general than the commonly used dynamic parallel distributed compensators for T-S fuzzy systems. For instance, the quadratically parameterized DPDC controller is described by

$$\begin{cases} \dot{x}_k = \sum_{i=1}^m \sum_{j=1}^m g_i(z) g_j(z) A_{k,ij} x_k + \sum_{i=1}^m g_i(z) B_{k,i} y, \\ u = \sum_{i=1}^m g_i(z) C_{k,i} x_k + D_{k,i} y. \end{cases}$$

Through a similar procedure to the T-S fuzzy plant conversion, the above DPDC controller can be rewritten in the controller LFT form (12)-(13). In fact, the DPDC controller is degenerated from the LFT gain-scheduled controller by specifying $B_{k0,f} = E_{x_k}$, $D_{k20,f} = E_u$ and imposing some structural constraints on the matrices C_{k0} , $D_{k00,f}$ and D_{k02} . This implies that a better performance can be achieved with the gain-scheduled controller over the DPDC.

3.2. Controller synthesis condition

By combining equations (7)-(9) and (12)-(13), we obtain the closed-loop system as below

$$\begin{bmatrix} \dot{x}_{cl} \\ q_{cl,s} \\ q_{cl,f} \\ e \end{bmatrix} = \begin{bmatrix} A_{cl} & B_{cl0,s} & B_{cl0,f} & B_{cl1} \\ C_{cl0,s} & D_{cl00,ss} & D_{cl00,sf} & D_{cl01,s} \\ C_{cl0,f} & D_{cl00,fs} & D_{cl00,ff} & D_{cl01,f} \\ C_{cl1} & D_{cl10,s} & D_{cl10,f} & D_{cl11} \end{bmatrix} \begin{bmatrix} x_{cl} \\ p_{cl,s} \\ p_{cl,f} \\ d \end{bmatrix}, \quad (14)$$

$$p_{cl,s} = dz(q_{cl,s}), \quad (15)$$

$$p_{cl,f} = \Theta_{cl} q_{cl,f}, \quad (16)$$

where $x_{cl} = \begin{bmatrix} x \\ x_k \end{bmatrix}$, $q_{cl,s} = u$, $p_{cl,s} = p_s$, $q_{cl,f} = \begin{bmatrix} q_f \\ q_{k,f} \end{bmatrix}$, $p_{cl,f} = \begin{bmatrix} p_f \\ p_{k,f} \end{bmatrix}$, $\Theta_{cl} = \begin{bmatrix} \Theta_f & 0 \\ 0 & \Theta_f \end{bmatrix}$ and the gain matrices of the closed-loop system are shown at the top of next page.

Based on the closed-loop system (14)-(16), we present the main result of this work in the following theorem.

Theorem 1. Given scalars $\gamma, s > 0$, if there exist positive definite matrices $R, S \in \mathbf{S}_+^n$, a diagonal matrix $L_s \in \mathbf{S}_+^{n_u}$, $L_f, J_f \in \mathbf{S}_+^{m(n+n_e+n_y)}$, $H_1 \in \mathbf{R}^{n_u \times n}$ and $\bar{H}_2 \in \mathbf{R}^{n_u \times n}$ such that

$$\begin{bmatrix} \mathcal{H}_{\bar{\Phi}}^T & 0 \\ 0 & I \end{bmatrix} \left[\begin{array}{ccc|c} RA_0^T + A_0 R & \star & \star & \star \\ -L_s B_{2,0}^T - \bar{H}_2 & -2L_s & \star & \star \\ L_f E_x^T + \Delta_x R & -\Delta_u L_s & -2L_f & \star \\ \hline C_{1,0} R & -D_{12,0} L_s & E_e L_f & -\gamma^2 I \\ \hline \bar{B}_{1,0}^T & 0 & \Delta_d^T & D_{11,0}^T \end{array} \right] \begin{bmatrix} \star \\ \star \\ \star \\ \star \end{bmatrix} \times \begin{bmatrix} \mathcal{H}_{\bar{\Phi}} & 0 \\ 0 & I \end{bmatrix} < 0, \quad (17)$$

$$\begin{bmatrix} \mathcal{H}_{\Gamma}^T & 0 \\ 0 & I \end{bmatrix} \left[\begin{array}{ccc|c} A_0^T S + S A_0 & \star & \star & \star \\ E_x^T S + J_f \Delta_x & -2J_f & \star & \star \\ \hline B_{1,0}^T S & \Delta_d^T J_f & -I & \star \\ \hline C_{1,0} & E_e & D_{11,0} & -\gamma^2 I \end{array} \right] \begin{bmatrix} \star \\ \star \\ \star \\ \star \end{bmatrix} \times \begin{bmatrix} \mathcal{H}_{\Gamma} & 0 \\ 0 & I \end{bmatrix} < 0, \quad (18)$$

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0, \quad (19)$$

$$\begin{bmatrix} L_f & I \\ I & J_f \end{bmatrix} > 0, \quad (20)$$

$$\begin{bmatrix} \frac{\bar{u}_\ell^2}{s^2} (\bar{H}_2)_\ell (H_1)_\ell \\ \star & R & I \\ \star & \star & S \end{bmatrix} \geq 0, \quad \forall \ell \in \mathbf{I}[1, n_u], \quad (21)$$

where the matrices $\mathcal{H}_{\bar{\Phi}}$ and \mathcal{H}_{Γ} denote matrices whose columns are the bases of $\text{Ker} \begin{bmatrix} B_{2,0}^T & I & \Delta_u^T & D_{12,0}^T \end{bmatrix}$ and $\text{Ker} \begin{bmatrix} C_{2,0} & E_y & D_{21,0} \end{bmatrix}$ respectively, then an n th-order gain-scheduled output feedback controller in the form of (12)-(13) will asymptotically stabilize the plant and render the H_∞ performance of the closed-loop system less than γ for any bounded disturbance $d \in \mathcal{W}_s$.

Proof. Using a quadratic Lyapunov function $V(x_{cl}) = x_{cl}^T P x_{cl}$, a diagonal matrix $\Lambda_1 > 0$ and a matrix $\Lambda_2 > 0$ commutable with Θ , i.e., $\Theta \Lambda_2 = \Lambda_2 \Theta$, the following Lyapunov condition

$$\begin{aligned} & \dot{V} + \frac{1}{\gamma^2} e^T e - d^T d + p_{cl,s}^T \Lambda_1 (q_{cl,s} - H x_{cl} - p_{cl,s}) \\ & + (q_{cl,s} - H x_{cl} - p_{cl,s})^T \Lambda_1 p_{cl,s} \\ & + p_{cl,f}^T \Lambda_2 (q_{cl,f} - p_{cl,f}) + (q_{cl,f} - p_{cl,f})^T \Lambda_2 p_{cl,f} < 0, \end{aligned}$$

$$\begin{aligned}
& \begin{bmatrix} A_{cl} & B_{cl0,s} & B_{cl0,f} & B_{cl1} \\ C_{cl0,s} & D_{cl00,ss} & D_{cl00,sf} & D_{cl01,s} \\ C_{cl0,f} & D_{cl00,fs} & D_{cl00,ff} & D_{cl01,f} \\ C_{cl1} & D_{cl10,s} & D_{cl10,f} & D_{cl11} \end{bmatrix} \\
&= \begin{bmatrix} A_0 & 0 & -B_{2,0} & E_x & 0 & B_{1,0} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta_x & 0 & -\Delta_u & 0 & 0 & \Delta_d \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_{1,0} & 0 & -D_{12,0} & E_e & 0 & D_{11,0} \end{bmatrix} + \begin{bmatrix} 0 & 0 & B_{2,0} \\ I & 0 & 0 \\ 0 & 0 & I \\ 0 & 0 & \Delta_u \\ 0 & I & 0 \\ 0 & 0 & D_{12,0} \end{bmatrix} \begin{bmatrix} A_k & B_{k0,f} & B_{k0,s} & B_{k2} \\ C_{k0} & D_{k00,f} & D_{k00,s} & D_{k02} \\ C_{k2} & D_{k20,f} & D_{k20,s} & D_{k22} \end{bmatrix} \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ C_{2,0} & 0 & 0 & E_y & 0 & D_{21,0} \end{bmatrix} \\
&:= \begin{bmatrix} \mathcal{A} & \mathcal{B}_{0,s} & \mathcal{B}_{0,f} & \mathcal{B}_1 \\ \mathcal{C}_{0,s} & \mathcal{D}_{00,ss} & \mathcal{D}_{00,sf} & \mathcal{D}_{01,s} \\ \mathcal{C}_{0,f} & \mathcal{D}_{00,fs} & \mathcal{D}_{00,ff} & \mathcal{D}_{01,f} \\ \mathcal{C}_1 & \mathcal{D}_{10,s} & \mathcal{D}_{10,f} & \mathcal{D}_{11} \end{bmatrix} + \begin{bmatrix} \mathcal{E}_0 \\ \mathcal{E}_{1,s} \\ \mathcal{E}_{1,f} \\ \mathcal{E}_2 \end{bmatrix} \Pi [\mathcal{G}_0 \mathcal{G}_{1,s} \mathcal{G}_{1,f} \mathcal{G}_2]
\end{aligned}$$

and the set inclusion condition

$$\{x_{cl} : x_{cl}^T P x_{cl} \leq s^2\} \subset \{x_{cl} : |H_\ell x_{cl}| \leq \bar{u}_\ell, \ell \in \mathbf{I}[1, n_u]\},$$

with $H = [H_1 \ H_2]$, will guarantee the stability and H_∞ performance of the closed-loop system in the existence of actuator saturation. Therefore, we have the Scaled Bounded Real Lemma as inequality

$$\begin{bmatrix} A_{cl}^T P + P A_{cl} & \star & \star & \star \\ B_{cl0,s}^T P + \Lambda_1 (C_{cl0,s} - H) & \left\{ \begin{array}{l} \Lambda_1 (D_{cl00,ss} - I) \\ + (D_{cl00,ss} - I)^T \Lambda_1 \end{array} \right\} & \star & \star \\ B_{cl0,f}^T P + \Lambda_2 C_{cl0,f} & \Lambda_2 D_{cl00,fs} + D_{cl00,sf}^T \Lambda_1 & \star & \star \\ B_{cl1}^T P & D_{cl01,s}^T \Lambda_1 & \star & \star \\ C_{cl1} & D_{cl10,s} & \star & \star \\ \left\{ \begin{array}{l} \Lambda_2 (D_{cl00,ff} - I) \\ + (D_{cl00,ff} - I)^T \Lambda_2 \end{array} \right\} & \star & \star & \star \\ D_{cl01,f}^T \Lambda_2 & -I & \star & \star \\ D_{cl10,f} & D_{cl11} & -\gamma^2 I & \star \end{bmatrix} < 0, \quad (22)$$

with the condition

$$\begin{bmatrix} \frac{\bar{u}_\ell^2}{s^2} & H_\ell \\ \star & P \end{bmatrix} \geq 0, \quad \ell \in \mathbf{I}[1, n_u]. \quad (23)$$

Taking equation (14) into consideration, the inequality (22) can be rewritten as

$$\Psi + \Gamma^T \Pi^T \Phi + \Phi^T \Pi \Gamma < 0, \quad (24)$$

where

$$\Psi = \begin{bmatrix} \mathcal{A}^T P + P \mathcal{A} & \star & \star & \star \\ \mathcal{B}_{0,s}^T P + \Lambda_1 (\mathcal{C}_{0,s} - H) & \left\{ \begin{array}{l} \Lambda_1 (\mathcal{D}_{00,ss} - I) \\ + (\mathcal{D}_{00,ss} - I)^T \Lambda_1 \end{array} \right\} & \star & \star \\ \mathcal{B}_{0,f}^T P + \Lambda_2 \mathcal{C}_{0,f} & \Lambda_2 \mathcal{D}_{00,fs} + \mathcal{D}_{00,sf}^T \Lambda_1 & \star & \star \\ \mathcal{B}_1^T P & \mathcal{D}_{01,s}^T \Lambda_1 & \star & \star \\ \mathcal{C}_1 & \mathcal{D}_{10,s} & \star & \star \\ \left\{ \begin{array}{l} \Lambda_2 (\mathcal{D}_{00,ff} - I) \\ + (\mathcal{D}_{00,ff} - I)^T \Lambda_2 \end{array} \right\} & \star & \star & \star \\ \mathcal{D}_{01,f}^T \Lambda_2 & -I & \star & \star \\ \mathcal{D}_{10,f} & \mathcal{D}_{11} & -\gamma^2 I & \star \end{bmatrix},$$

$$\Gamma = [\mathcal{G}_0 \ \mathcal{G}_{1,s} \ \mathcal{G}_{1,f} \ \mathcal{G}_2 \ 0],$$

$$\Phi = [\mathcal{E}_0^T P \ \mathcal{E}_{1,s}^T \Lambda_1 \ \mathcal{E}_{1,f}^T \Lambda_2 \ 0 \ \mathcal{E}_2^T].$$

Then, applying Elimination Lemma, inequality (24) can be expressed as

$$\begin{cases} \mathcal{N}_\Phi^T \Psi \mathcal{N}_\Phi < 0 \\ \mathcal{N}_\Gamma^T \Psi \mathcal{N}_\Gamma < 0 \end{cases}, \quad (25)$$

where \mathcal{N}_Γ and \mathcal{N}_Φ are the null matrices of Γ and Φ , respectively. Note that Φ satisfies $\Phi = \tilde{\Phi} T$ with

$$\tilde{\Phi} = [\mathcal{E}_0^T \ \mathcal{E}_{1,s}^T \ \mathcal{E}_{1,f}^T \ 0 \ \mathcal{E}_2^T], \quad T = \begin{bmatrix} P & 0 & 0 & 0 & 0 \\ 0 & \Lambda_1 & 0 & 0 & 0 \\ 0 & 0 & \Lambda_2 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}.$$

Consequently, inequality (25) can be further written as

$$\begin{cases} \mathcal{N}_{\tilde{\Phi}}^T \tilde{\Psi} \mathcal{N}_{\tilde{\Phi}} < 0 \\ \mathcal{N}_{\tilde{\Gamma}}^T \tilde{\Psi} \mathcal{N}_{\tilde{\Gamma}} < 0 \end{cases}, \quad (26)$$

where $\mathcal{N}_{\tilde{\Phi}}$ is the null matrix of $\tilde{\Phi}$ and $\tilde{\Psi} = T^{-T} \Psi T^{-1}$.

Furthermore, we have

$$\begin{aligned} \tilde{\Phi} &= [\mathcal{E}_0^T \mathcal{E}_{1,s}^T \mathcal{E}_{1,f}^T 0 \mathcal{E}_2^T] \\ &= \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ B_{2,0}^T & 0 & I & \Delta_u^T & 0 & 0 \\ D_{12,0}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \tilde{\Gamma} &= [\mathcal{G}_0 \mathcal{G}_{1,s} \mathcal{G}_{1,f} \mathcal{G}_2 \ 0] \\ &= \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ C_{2,0} & 0 & 0 & E_y & 0 & D_{21,0} \end{bmatrix}. \end{aligned}$$

Then, the bases of the kernel space of $\tilde{\Phi}$ and $\tilde{\Gamma}$ can be derived as follows

$$\begin{aligned} \mathcal{N}_{\tilde{\Phi}}^T &= \begin{bmatrix} \mathcal{H}_{\tilde{\Phi}_1}^T & 0 & \mathcal{H}_{\tilde{\Phi}_2}^T & 0 & \mathcal{H}_{\tilde{\Phi}_3}^T & 0 & 0 & \mathcal{H}_{\tilde{\Phi}_4}^T \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}, \\ \mathcal{N}_{\tilde{\Gamma}}^T &= \begin{bmatrix} \mathcal{H}_{\tilde{\Gamma}_1}^T & 0 & 0 & \mathcal{H}_{\tilde{\Gamma}_2}^T & 0 & \mathcal{H}_{\tilde{\Gamma}_3}^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}. \end{aligned}$$

By assuming $\Lambda_1^{-1} = L_s$ and

$$\begin{aligned} P &= \begin{bmatrix} S & N \\ N^T & \star_1 \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} R & M \\ M^T & \star_2 \end{bmatrix}, \\ \Lambda_2 &= \begin{bmatrix} J_f & V_f \\ V_f^T & K_f \end{bmatrix}, \quad \Lambda_2^{-1} = \begin{bmatrix} L_f & U_f \\ U_f^T & Q_f \end{bmatrix}, \end{aligned}$$

we have the representations of $\tilde{\Psi}$ and Ψ printed at the top of next page. Besides, assuming $\bar{H}_2 = H_1 R + H_2 M^T$ and deriving from inequality (26), we can arrive at the LMIs (17)-(18).

The inequality (19) is a equivalent condition of $P > 0$ and $P^{-1} > 0$. The inequality (20) is a equivalent condition of $\Lambda_2 > 0$ and $\Lambda_2^{-1} > 0$. To verify inequality (21), we choose the invertible matrix $Z = \text{diag} \left\{ 1, \begin{bmatrix} R & M \\ I & 0 \end{bmatrix} \right\}$ and multiply (23) by Z from left and Z^T from right side to get

$$\begin{bmatrix} \frac{\bar{u}_\ell^2}{s^2} (H_1 R + H_2 M^T)_\ell (H_1)_\ell \\ \star & R & I \\ \star & \star & S \end{bmatrix} \geq 0,$$

which is equivalent to the inequality (21). \square

Remark 1. To minimize the closed-loop performance, the feasibility condition (17)-(21) can also be solved by the following optimization problem

$$\begin{aligned} \min_{R, S, L_s, J_f, L_f, \bar{H}_2, H_1} \quad & \gamma^2 \\ \text{subject to} \quad & (17) - (21) \end{aligned}, \quad (27)$$

Based on Theorem 1, the gain matrices of the gain-scheduled output feedback controller can be determined via a constructive procedure as follows:

Step 0 : Obtain $R, S, L_s, L_f, J_f, \bar{H}_2$ and H_1 by solving the optimization problem (27) or solving the feasibility problem of LMIs (17)-(21).

Step 1 : Select M, N, U_f, V_f matrices such that $MN^T = I - RS$ and $U_f V_f^T = I - L_f J_f$.

Step 2 : Compute H_2 by $H_2 = (\bar{H}_2 - H_1 R)M^{-T}$.

Step 3 : Calculate Π from (24) as an LMI feasibility problem to obtain the controller gains.

Remark 2. In order to find controller gains of reasonable size, some additional constraints might be imposed on the decision variables R, S, L_s, L_f, J_f such as $R^T R < n^2 \tau^2 I$, $S^T S < n^2 \tau^2 I$, $L_s^T L_s < n_s^2 \tau^2 I$, $L_f^T L_f < m^2 (n + n_e + n_y)^2 \tau^2 I$, and $J_f^T J_f < m^2 (n + n_e + n_y)^2 \tau^2 I$ which can also be transformed into LMI conditions easily. τ is a pre-specified constant to restrict the magnitude of gain matrices.

4. Simulation example

To illustrate the effectiveness of the proposed approach, we consider the problem of balancing an inverted pendulum on a cart. The dynamics of the inverted pendulum can be described by

$$\dot{x}_1 = x_2, \quad (28)$$

$$\dot{x}_2 = \frac{g \sin(x_1) - a M_1 l x_2^2 \sin(2x_1)/2 - a \cos(x_1) \text{sat}(u)}{4l/3 - a M_1 l \cos^2(x_1)}, \quad (29)$$

where x_1 represents the angle of the pendulum from the vertical position and x_2 is the angular velocity. Besides, $g = 9.8m/s^2$ is the gravity constant, M_1 is the mass of the pendulum, M_2 is the mass of the cart, $a = 1/(M_1 + M_2)$ is introduced for brevity, $2l$ is the length of the pendulum, and $\text{sat}(u)$ is the force ap-

$$\tilde{\Psi} = \begin{bmatrix} RA_0^T + A_0R & \star & \star & \star & \star & \star & \star \\ M^T A_0^T & 0 & \star & \star & \star & \star & \star \\ -L_s B_{2,0}^T - H_1 R - H_2 M^T - H_1 M - H_2 \star_2 & -2L_s & \star & \star & \star & \star & \star \\ \bar{L}_f E_x^T + \Delta_x R & \Delta_x M & -\Delta_u L_s & -2L_f & \star & \star & \star \\ U_f^T E_x^T & 0 & 0 & -2U_f^T & -2Q_f & \star & \star \\ B_{1,0}^T & 0 & 0 & \Delta_d^T & 0 & -I & \star \\ C_{1,0} R & C_{1,0} M & -D_{12,0} L_s & E_e L_f & E_e U_f & D_{11,0} & -\gamma^2 I \end{bmatrix}$$

$$\Psi = \begin{bmatrix} A_0^T S + S A_0 & \star & \star & \star & \star & \star & \star \\ N^T A_0 & 0 & \star & \star & \star & \star & \star \\ -B_{2,0}^T S - \Lambda_1 H_1 - B_{2,0}^T N - \Lambda_1 H_2 & -2\Lambda_1 & \star & \star & \star & \star & \star \\ E_x^T S + J_f \Delta_x & E_x^T N & -J_f \Delta_u & -2J_f & \star & \star & \star \\ V_f^T \Delta_x & 0 & -V_f^T \Delta_u & -2V_f^T & -2K_f & \star & \star \\ B_{1,0}^T S & B_{1,0}^T N & 0 & \Delta_d^T J_f & \Delta_d^T V_f & -I & \star \\ C_{1,0} & 0 & -D_{12,0} & E_e & 0 & D_{11,0} & -\gamma^2 I \end{bmatrix}$$

plied to the cart. In the following study, we choose $M_1 = 2.0\text{kg}$, $M_2 = 8.0\text{kg}$, $l = 0.5\text{m}$ and $\bar{u} = 150\text{N}$.

In order to approximate the above system by a T-S fuzzy model in the form of (1), d is selected as a two-dimensional vector containing a disturbance force on the cart and a measurement noise as its elements, y is the output of the angle sensor, and e is the summation of weighted u and x_1 . The measurement output signal y is chosen as the premise variable z . Given the operating range $x_1 \in (-\pi/4, \pi/4)$, we have

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3-aml} & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ \frac{4g/\pi \sin(\pi/4)}{4l/3-aml \cos^2(\pi/4)} & 0 \end{bmatrix},$$

$$B_{1,1} = \begin{bmatrix} 0 & 0 \\ \frac{-a}{10(4l/3-aml)} & 0 \end{bmatrix}, \quad B_{1,2} = \begin{bmatrix} 0 & 0 \\ \frac{-a \cos(\pi/4)}{4l/3-aml \cos^2(\pi/4)} & 0 \end{bmatrix},$$

$$B_{2,1} = \begin{bmatrix} 0 \\ \frac{-a}{4l/3-aml} \end{bmatrix}, \quad B_{2,2} = \begin{bmatrix} 0 \\ \frac{-a \cos(\pi/4)}{4l/3-aml \cos^2(\pi/4)} \end{bmatrix},$$

$$C_1 = [1 \ 0], \quad C_2 = [1 \ 0],$$

$$D_{12} = 0.01, \quad D_{21} = [0 \ 0.01].$$

Rule 1 :

IF y is about 0

$$\text{THEN} \quad \begin{cases} \dot{x} = A_1 x + B_{1,1} d + B_{2,1} \text{sat}(u) \\ e = C_1 x + D_{12} \text{sat}(u) \\ y = C_2 x + D_{21} d \end{cases},$$

Rule 2 :

IF y is about $\pm \pi/4$

$$\text{THEN} \quad \begin{cases} \dot{x} = A_2 x + B_{1,2} d + B_{2,2} \text{sat}(u) \\ e = C_1 x + D_{12} \text{sat}(u) \\ y = C_2 x + D_{21} d \end{cases},$$

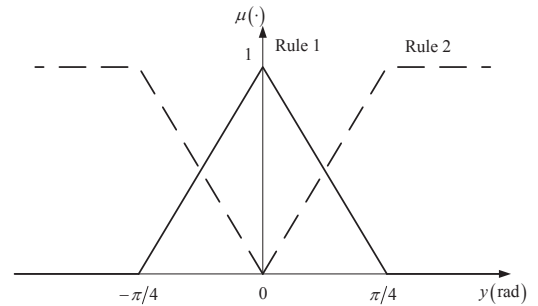


Fig. 1. Membership Functions

The membership functions for Rules 1 and 2 are chosen to be triangular membership functions as

shown in Fig. 1. $\theta_{f,i}$ can be calculated according to

$$\theta_{f,i} = \frac{\mu_i(y)}{\sum_{i=1}^2 \mu_i(y)}.$$

By solving the optimization problem (27), the optimal value $\gamma = 0.0756$ can be achieved for $s = 6$, and $\gamma = 0.1559$ for $s = 18$. In order to investigate the influence of pre-specified constants s and τ , three groups of synthesis results are given in Table 1 and the corresponding controller gains are provided on the top of next page.

Table 1
Synthesis Results

	s	τ	γ
Controller 1	6	1000	0.0814
Controller 2	6	200	0.1075
Controller 3	18	1000	0.3880

During the synthesis process, it is observed that for the same energy bound of disturbance s , larger magnitude constraint τ would achieve better performance γ but render higher controller gains, so a tradeoff is needed for controller design in practice. Moreover, smaller energy bound of disturbance s can render better performance γ with the same magnitude constraint τ , so a priori knowledge of the disturbance is also important for achieving better performance.

Then, two simulations are conducted to verify the stability and disturbance attenuation performance of these three controllers, respectively. In the first simulation, the initial angle is set to 45° , and no disturbance is added. The state responses of the closed-loop system are shown in Fig. 2 and the controller outputs u and $\text{sat}(u)$ are shown in Fig. 3. It is clear that the nonlinear inverted pendulum is stabilized by all gain-scheduled controllers.

The second simulation examines controlled performance of disturbance suppression and the disturbance d is chosen to be a pulse force starting at 1sec and ending at 3sec with the disturbance magnitude of 355N . Figs. 4 and 5 depict the state responses and the controller outputs with disturbance d , respectively. It is observed that the gain-scheduled output feedback controllers succeed in rejecting the disturbance as well as keeping the closed-loop system stable. Nevertheless, controllers 2 and 3 generate larger transient responses with bigger control forces due to their inferior performance to controller 1. This successful application shows the validity of the proposed method in this work.

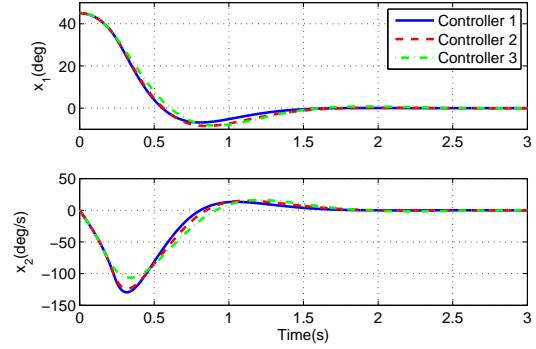


Fig. 2. State responses with initial angle $x_1(0) = \pi/4$

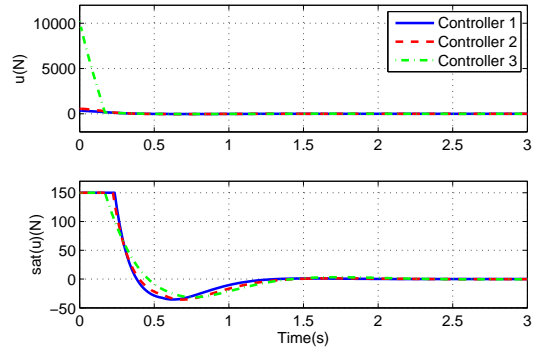


Fig. 3. Force on the cart with initial angle $x_1(0) = \pi/4$

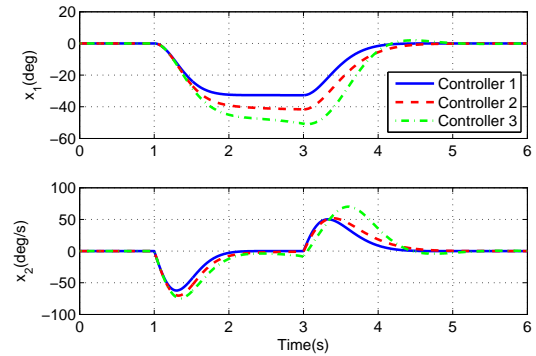
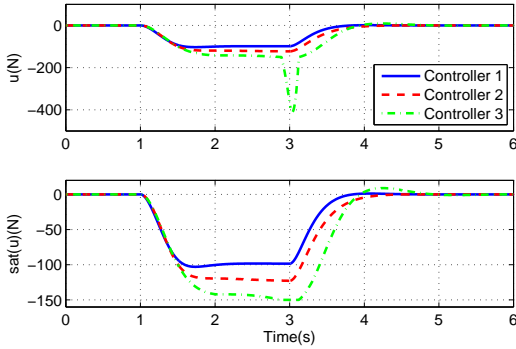


Fig. 4. State responses with a pulse disturbance d

5. Conclusions

In this article, a systematic synthesis method is proposed for gain-scheduled output feedback controller design for T-S fuzzy systems with actuator saturation. To the best of our knowledge, this is the first attempt to solve the controller design problem of T-

$$\begin{aligned}
\Pi_1 = & \begin{bmatrix} -65.4588 & -23.9431 & 0.0926 & 3.9424 & 1.5865 & -6.6291 & -8.2663 & -10.5407 \\ -220.2901 & -106.5108 & 0.3804 & 23.9237 & 6.1831 & -8.1466 & -31.3295 & -95.9381 \\ 0.1643 & -0.0404 & 0.0000 & -0.1350 & -0.0230 & -0.0483 & -0.0093 & -0.3505 \\ -5.3984 & 1.3956 & -0.0020 & -1.2561 & -0.2501 & -2.9106 & -0.5604 & 9.6818 \\ 0.1643 & -0.0404 & 0.0000 & 0.0169 & 0.0031 & -0.2005 & -0.0355 & -0.3505 \\ 6.3726 & -1.6351 & 0.0021 & 0.9625 & 0.1933 & 2.2354 & 0.4307 & -11.7385 \\ -210.9741 & 54.0049 & 0.9290 & -33.8299 & -6.7463 & -77.3320 & -14.8761 & 435.3788 \end{bmatrix} \\
\Pi_2 = & \begin{bmatrix} -63.8092 & -25.5385 & 0.5697 & 2.6084 & 0.4516 & -12.6843 & -6.7052 & -5.8392 \\ -156.8813 & -78.8726 & 1.6359 & 12.5534 & 0.8017 & -11.5590 & -15.5954 & -43.9174 \\ 0.2140 & -0.0755 & 0.0001 & -0.6125 & -0.1046 & -0.0335 & -0.0065 & -0.5839 \\ -3.2115 & 1.3518 & -0.0141 & -1.0954 & -0.2189 & -3.5340 & -0.6762 & 7.5001 \\ 0.2139 & -0.0755 & 0.0001 & 0.0181 & 0.0032 & -0.6642 & -0.1144 & -0.5836 \\ 3.5211 & -1.4608 & 0.0143 & 0.7124 & 0.1496 & 3.0783 & 0.5932 & -8.3267 \\ -112.6738 & 46.6950 & 0.5305 & -24.3981 & -5.0827 & -110.3414 & -21.0991 & 312.5316 \end{bmatrix} \\
\Pi_3 = & \begin{bmatrix} -40.3497 & -18.6563 & 0.0028 & 0.2508 & 5.0318 & -1.0158 & -17.2637 & -7.0804 \\ -131.4672 & -82.4518 & 0.0115 & 4.8569 & 19.1502 & 9.2371 & -67.8685 & -58.3868 \\ 0.1968 & -0.0490 & 0.0000 & -0.3046 & -0.0705 & -0.0345 & -0.0090 & -0.3819 \\ -2.8636 & 0.7148 & -0.0001 & -1.1068 & -0.2861 & -1.9251 & -0.5008 & 4.4680 \\ 0.1966 & -0.0490 & 0.0000 & 0.0423 & 0.0108 & -0.3814 & -0.0903 & -0.3816 \\ 4.3018 & -1.0732 & 0.0003 & 0.2777 & 0.0747 & 0.5343 & 0.1445 & -7.2423 \\ -144.8399 & 36.1313 & 0.9884 & -15.5568 & -4.0977 & -48.7062 & -12.6553 & 293.4149 \end{bmatrix}
\end{aligned}$$

Fig. 5. Force on the cart with a pulse disturbance d

S fuzzy model by utilizing the powerful LFT mechanism within the framework of robust control theory. Based on the model transformation method in this paper, many powerful existing robust techniques can be employed to solve the control problems of T-S fuzzy systems conveniently. To this end, the saturation control synthesis problem is cast as a convex optimization in terms of LMIs and can be solved efficiently by interior-point algorithms. If the solution to the LMIs is feasible, a gain-scheduled output feedback controller can be constructed to guarantee the asymptotic stabil-

ity and regional H_∞ performance of the closed-loop system. The example of balancing of an inverted pendulum on a cart illustrates the effectiveness of the proposed approach.

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